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The Rev. R. Sheepshanks, Vice-President, in the Chair.

Professor Haquin Selander, Astronomer Royal at Stockholm, and the Baron Fabian Jacob Wrede, Director of the Academy of Sciences at Stockholm, were balloted for, and duly elected Associates of the Society.

John George Cockburn Curtis, Esq. of the Hydrographic Office, Admiralty, and Arthur Kett Barclay, Esq. of Norbury, Croydon, were balloted for, and duly elected Fellows of the Society.

The following communications were read:—

I. On the Flexure of a uniform Bar supported by a number of equal Pressures applied at equi-distant points, and on the Positions proper for the Applications of these Pressures, in order to prevent any sensible Alteration of the Length of the Bar by small Flexure. By G. B. Airy, Esq. Astronomer Royal.

The author observes that this communication is, in reality, merely an auxiliary in a long series of troublesome researches, and ought, in strictness, to accompany the account of those researches, wherever it shall be ultimately placed. He thought, however, that it would be agreeable to the Society, both because it treats of mathematical formulæ and physical experiments which are familiar to many members of the Society, and also because its results bear in some degree upon the appreciation of one of our possessions, which is, perhaps, more creditable to us than any other, viz. the Standard Scale of the Society.

In the use of Standards of Length, two points have gradually attracted more and more of attention — the application of supports in such a manner as to produce no irregularities of flexure, and the application of such supports as will permit the standard freely to yield to the expansive or contractive effects of temperature. Since these points were insisted on by Captain Kater and Mr. Baily, the principle adopted in the support of standards has been to sustain them upon two rollers in a definite position. In whatever way a bar be supported, flexure must be tolerated; and the only points aimed at are, that it shall produce no sensible effects on the measured length, or that it shall always be the same.

Several bars in use are notched at the ends to the centre of their thickness, and the defining points are in the axis of the bar; and in these cases there is good reason to believe that the effects of any flexure are nearly insensible. In the Standard Scale of this Society, the divisions are upon the upper surface, and the interval between these is not unaffected by flexure. In this case there may be some fear of the bar acquiring a permanent flexure, but this may be checked by an examination of the divisions which Mr. Baily caused to be cut upon the other sides of the cylinder. Still it is necessary to be assured that, as far as the best theories of flexure enable us to judge, the upper surface is not sensibly lengthened or shortened.

Very simple considerations will convince us that bars supported on two rollers, placed at the distance of one-fourth of the length from each end, must have the length of their upper surface necessarily elongated. The ill effects of this elongation are nearly obviated in the geodetic standards, which are notched to the centre; and may be considered as discoverable in the Society's scale by the use of the divisions on the opposite side. But it was the wish of the members of the Committee of Superintendence of the National Standards, not only to place the divisions in or near to the axis of the bar, by which means the *effect* of flexure would be obviated, but also to make the actual flexure as small as possible by a proper choice of the points of support. It was also recognised as a desirable principle that the bar should be subjected to as little strain as possible, and, therefore, that it should be supported at numerous points.

Great facility is given to the arrangements for supporting a bar with definite pressures applied at special points, by the use of levers. Thus, if any portion of the bar rest upon two rollers which are placed at the ends of a lever, and if the fulcrum of a lever (whether moveable or not) be in its centre, the pressures upwards produced by these rollers will necessary be equal. If there be another such lever, and if the fulcra of this and the former be upon the extremities of a third lever, and if its fulcrum be at its centre, then the pressures upwards produced by the four rollers will be equal. By this arrangement of rollers and levers one half of the bar may be supported. If another similar system be applied to support the other half of the bar, the pressures produced by its four rollers will also be equal among themselves; and if the bar be laid symmetrically upon them, all the individual pressures will be equal In this manner, by altering the lengths of the arms of the levers, any number of pressures, bearing any proportion to each other, and applied at any points whatever, may be produced. And the levers may be so arranged as to occupy a very small space.

Mr. Baily decided on the application of eight rollers for the

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support of the national standard; and the author undertook to investigate the positions of the rollers which, supposing the pressures and the intervals equal, would so sustain the bar that its surface should not be sensibly lengthened. In the progress of the investigation he was struck with an analogy in the results for two, for four, and for eight rollers, which seemed to indicate that the process could easily be generalised; and, on trying it in the general form, it proved to be exactly what he expected. The author goes on to describe the nature of the investigation, which is as follows:—

It is first assumed, that the flexure is so small that the mere curvature of a neutral line will not produce a sensible alteration in its length; that the extension of a surface is proportional to the momentum of the bending force; and that, when the momenta are equal, the extension produced by a bending force downwards, and the contraction produced by a bending force upwards, are equal. The method of applying these assumptions is to estimate the curvature at any point, by conceiving the section of the bar at that point, and the whole of the bar on one side of that point, to be held perfectly firm; and then to use the algebraic sum of the momentum of the weight of the remaining part of the bar downwards, and of the momentum of the supporting pressures under that part of the bar upwards, as the representation of the molecular extension in the neighbourhood of that section.

From this statement it will easily be seen that the curvature of the bar is expressed by a discontinuous function. In shifting our ideal section gradually from one support to another, the momentum produced by the weight of the portion of the bar increases continuously; as also does the momentum of the pressures of the supports; and, therefore, through that portion of the bar the change of curvature is continuous. But on passing a new support, although the change in the momentum of the weight of the portion of the bar and the change in the momentum of the pressures of the former supports are both continuous, yet there is now suddenly introduced a new pressure; and this interrupts the continuity of the change. From this consideration it appears that, between two supports, the whole effect of molecular extension and contraction, as produced by curvature, is to be found by integration; but, that, when this integration is performed, the whole of the various parts can only be combined by summation.

These remarks are common to every method of supporting a bar. But when the supporting pressures are all equal, and their points of application are equally distant, the summation to which allusion is made can, for all the intervals between the supports, be effected by the processes of the calculus of differences. For the parts exterior to the terminal supports, these processes will not apply.

The author then proceeds with the mathematical investigation of the general expression for the extension of surface of the portion of the bar lying between any two of its contiguous supports.

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If W be the weight of the bar, and b the length between the nth and the $n + 1$th support, a the whole length of the bar, and m the whole number of supports (placed at equal distances), he proves that the extension of surface of that portion of the bar lying between the nth and $n + 1$th support will be represented by

$$\frac{\mathrm{W}\, b}{6\, a}\left\{3 \,.\, n \,.\, \overline{n - 1} \,.\, b^2 + b^2 + 6 \,.\, \overline{n - \tfrac{1}{2}} \,.\, b\, c + 3\, c^2\right\}$$

$$- \frac{b^2\, \mathrm{W}}{2\, m} \,.\, n \,.\, \overline{n - 1} - \frac{b^2\, \mathrm{W}}{2\, m} \,.\, n.$$

The sum of all the extensions is then taken from $n = 1$ to $n = m$, which gives the whole extension between the 1st and the mth, or last support.

If, also, c be the length of bar projecting beyond each of the terminal supports, and e the distance from each end of the bar at which the defining points are marked, then the additional extensions at both ends of the bar of the length $c - e$, will, by integration, be found to be represented by $\frac{c^3 - e^3}{6\, a}$ W. Adding this to the former expression, and remembering that

$$c = \frac{a}{2} - \overline{m - 1} \,.\, \frac{b}{2},$$

the expression for the whole extension of the bar between the defining points reduces itself to the very simple form

$$\frac{\mathrm{W}}{24\, a}\left\{a^3 - 8\, e^3 - (m^2 - 1)\; a\; b^2.\right\}$$

And if this be made $= 0$

$$b = \frac{a}{\sqrt{m^2 - 1}} \sqrt{1 - \frac{8\, e^3}{a^3}}.$$

II. An Account of some Ancient Astronomical Tables in the Library of the Reverend Charles Turnor, A.M., F.R.S., &c. By Mr. Richard Harris, Assistant-Secretary of the Society.

One of the volumes containing these tables is without date or title, but from its containing a calendar for the year 1349, there can be little doubt of that portion, at least, having been written in the early part of the fourteenth century. The other volume contains a calendar for the year 1347, and bears the title *Tabulæ Mediorum Motuum omnium Planetarum.* From an inscription on the fly-leaf it appears that it once belonged to the fraternity of Tolerants, or Grey Friars, at Babewell, near Bury St. Edmund's. The tables, which form the principal portion of the volume, were computed for the meridian of Oxford, and bear the general title "Almanak editum Oxoniæ:" they appear to have been chiefly formed from the famous Alphonsine Tables; the requisite correction being applied throughout for the difference of meridians, and some trifling alterations made in the method of arrangement to suit the